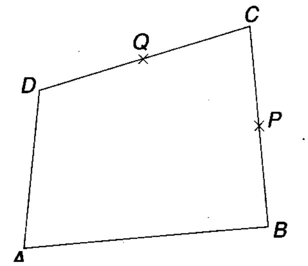
1. In the figure, $ABCD$ is a quadrilateral. $P$ and $Q$ are the midpoints of $BC$ and $CD$ respectively. Express each of the following sums as a single vector.

   (a) $\overrightarrow{AB} + \overrightarrow{BQ} + \overrightarrow{QP}$

   (b) $\overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DB}$

   (c) $\overrightarrow{AQ} + \overrightarrow{QB} + \overrightarrow{BD}$

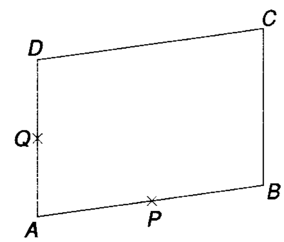



2. $ABCD$ is a parallelogram. $P$ and $Q$ are the midpoints of $AB$ and $AD$ respectively. Show that

   (a) $\overrightarrow{AP} + \overrightarrow{AQ} = \frac{1}{2}\overrightarrow{AC}$,

   (b) $\overrightarrow{PC} + \overrightarrow{QC} = \frac{3}{2}\overrightarrow{AC}$,

   (c) $\overrightarrow{PD} + \overrightarrow{QB} = \frac{1}{2}\overrightarrow{AC}$.

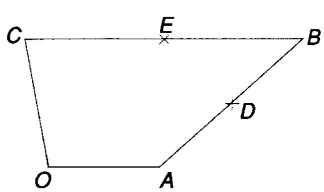



3. $OAB$ is a triangle and $M$ is the midpoint of $AB$. Express $\overrightarrow{AM}$ and $\overrightarrow{OM}$ in terms of $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

4. $OABC$ is a trapezium with $\overrightarrow{CB} = 3\overrightarrow{OA}$. The points $D$ and $E$ are midpoints of $AB$ and $BC$.

   (a) Express $\overrightarrow{OD}$, $\overrightarrow{OE}$ and $\overrightarrow{DE}$ in terms of $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

   (b) Express $\overrightarrow{OD}$, $\overrightarrow{OE}$ and $\overrightarrow{DE}$ in terms of $\overrightarrow{OA}$ and $\overrightarrow{OC}$.




5. $OABC$ is a parallelogram. Express the sum of the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ in terms of a single vector.

6. $ABCD$ is a rectangle. Given that $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{BC} = \mathbf{q}$, express in terms of $\mathbf{p}$ and $\mathbf{q}$,

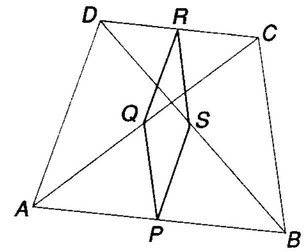
   (a) $\overrightarrow{AC}$,

   (b) $\overrightarrow{DB}$,

   (c) $\overrightarrow{BX}$, where $X$ is the midpoint of $CD$.

   Given that $|\mathbf{p}| = 2|\mathbf{q}|$ and $\mathbf{q}$ is a unit vector, evaluate $|\overrightarrow{BX}|$.

7. In the figure, $P$, $Q$, $R$ and $S$ are the mid-points of the sides $AB$, $AC$, $DC$, $DB$ of the quadrilateral $ABCD$. Show that $PQRS$ is a parallelogram.




*8. $ABCDEFGH$ is a regular octagon and $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{BC} = \mathbf{q}$. Express $\overrightarrow{AH}$ in terms of $\mathbf{p}$ and $\mathbf{q}$ and show that $\overrightarrow{AE} + \overrightarrow{BH} + \overrightarrow{CG} + \overrightarrow{DF} = 2(2 + \sqrt{2})(\mathbf{q} - \sqrt{2}\mathbf{p})$.

9. $ABCDEF$ is a regular hexagon. Express in terms of a single vector the sum of the vectors

   **(a)** $\overrightarrow{AB}, \overrightarrow{AE}$, **(b)** $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AE}, \overrightarrow{AF}$,

   **(c)** $\overrightarrow{AB}, \overrightarrow{AF}$, **(d)** $4\overrightarrow{AB}, 2\overrightarrow{AC}, \overrightarrow{AD}, \overrightarrow{AE}, 5\overrightarrow{AF}$. *(C)*

10. $ABCDEF$ is a regular hexagon. Given that $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{BC} = \mathbf{q}$, express the following in terms of one, or both, $\mathbf{p}$ and $\mathbf{q}$.

   **(a)** $\overrightarrow{AD} + \overrightarrow{BE}$ **(b)** $\overrightarrow{BD} + \overrightarrow{CE}$

**Exercise 23.1** *(p. 481)*

**1.** **(a)** $\overrightarrow{AP}$ **(b)** $\overrightarrow{AB}$ **(c)** $\overrightarrow{AD}$ **3.** $\frac{1}{2}(\overrightarrow{OB} - \overrightarrow{OA}), \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$

**4.** **(a)** $\frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB}), -\frac{3}{2}\overrightarrow{OA} + \overrightarrow{OB}, -2\overrightarrow{OA} + \frac{1}{2}\overrightarrow{OB}$

**(b)** $\frac{1}{2}\overrightarrow{OC} + 2\overrightarrow{OA}, \overrightarrow{OC} + \frac{3}{2}\overrightarrow{OA}, \frac{1}{2}\overrightarrow{OC} - \frac{1}{2}\overrightarrow{OA}$

**5.** $2\overrightarrow{OB}$ **6.** **(a)** $\mathbf{p} + \mathbf{q}$ **(b)** $\mathbf{p} - \mathbf{q}$ **(c)** $\mathbf{q} - \frac{1}{2}\mathbf{p}$; $\sqrt{2}$ units

**8.** $\mathbf{q} - \sqrt{2}\,\mathbf{p}$ **9.** **(a)** $\overrightarrow{AD}$ **(b)** $2\overrightarrow{AD}$ **(c)** $\frac{1}{2}\overrightarrow{AD}$ **(d)** $5\frac{1}{2}\overrightarrow{AD}$

**10.** **(a)** $4\mathbf{q} - 2\mathbf{p}$ **(b)** $3(\mathbf{q} - \mathbf{p})$